Killing the Fraction

approach for solving rational equations

Most textbook presents the ‘polished apple’ approach to solving rational equations, like this one:

$$\frac{1}{2x+10} = \frac{8}{x^2-25} - \frac{2}{x-5}$$

That method is to find the LCD of all the denominators in the problem, then multiply on both sides (or multiply each term), which will eliminate all fractions, reducing the problem to a previous type of equation.

The “Killing the Fraction” method doesn’t actually find the LCD directly, but looks at each fraction, one by one then ‘kills’ that fraction.

1. Either method requires that we first factor all denominators completely. So, after doing that we get . . .

$$\frac{1}{2(x+5)} = \frac{8}{(x-5)(x+5)} - \frac{2}{x-5}$$

2. Now focus on ‘killing’ each fraction, one by one, starting with

   a. the first fraction $\frac{1}{2(x+5)}$ by multiplying by its denominator of 2(x+5). But this has to be multiplied on each term. So, we get . . .

$$2(x+5)\frac{1}{2(x+5)} = 2(x+5)\frac{8}{(x-5)(x+5)} - 2(x+5)\frac{2}{x-5}$$

   This will clearly cancel the entire denominator on that first fraction. It also might cancel some or all of other fractions throughout the problem. In this case, the (x + 5) also cancels on the second fraction, leaving the factor of 2, but nothing cancels on the third (but we didn’t expect any cancellation to happen either). So, the problem reduces to:

$$1 = \frac{2 \times 8}{x-5} - \frac{2(x+5)\times 2}{x-5}$$

   which simplifies to . . .

$$1 = \frac{16}{x-5} - \frac{4(x+5)}{x-5}$$

   b. Now, look at the next fraction on the second term. We will multiply all terms, including the first term, which is no longer a fraction by x – 5. After doing this we would get . . .

$$(x-5)\times 1 = (x-5)\frac{16}{x-5} - (x-5)\frac{4(x+5)}{x-5}$$

   Again, this will clearly ‘kill’ the denominator on the second term (why we picked x – 5), but it also ‘kills’ the denominator on the third term, so we get . . .

$$x - 5 = 16 - 4(x+5)$$

   This reduces to $x - 5 = 16 - 4x - 20$ then, isolating the unknown, 5x = 1, so $x = \frac{1}{5}$

   Since the original equations had factors of (x – 5) and (x + 5) in the denominators, then x cannot equal 5 or -5 or else we would be dividing by zero.

   Since our solution is neither of these, we keep the solution as $\left\{\frac{1}{5}\right\}$.